Calculation of Marginal CO_2 Emissions Allowances Operational Cost for Hydro-Dominated Power Systems

Steffen Rebennack

Assistant Professor of Operations Research
Colorado School of Mines
Division of Economics & Business

www.rebennack.net
srebenna@mines.edu

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Collaborators

Panos M. Pardalos

Mario V.F. Pereira

Bruno Flach

Outline

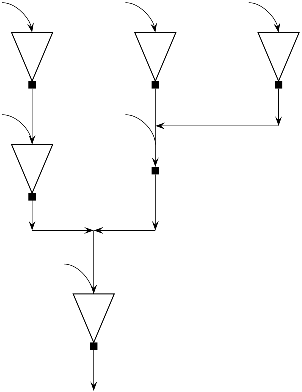
Hydro-Thermal Power Systems

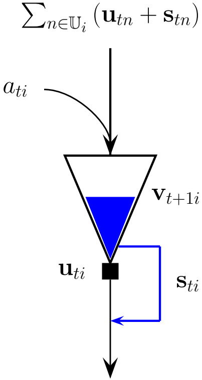
Figure: Itaipu, Brazil

Figure: Coal plant

Figure: Gas plant

Water Balance





$$\mathbf{v}_{t+1i} = v_{ti} - \mathbf{u}_{ti} - \mathbf{s}_{ti} + \sum_{n\in\mathbb{U}_i}(\mathbf{u}_{tn} + \mathbf{s}_{tn}) + a_{ti}$$

The World is Uncertain!?

...linear programming methods (to) be extended to include the case of uncertain demands for the problem of optimal allocation of a carrier fleet to airline routes to meet an anticipated demand distribution...

George B. Dantzig
Linear Programming under Uncertainty
Management Science, 1:3 & 4, 197–206, 1955

What Exactly is Uncertain?

Such an energy system is subject to different uncertainties:

- stochastic fuel prices,
- stochastic electricity demand,
- stochastic (water) inflows,

and in the liberalized market in addition also:

- stochastic electricity spot prices,
- stochastic CO2 prices.

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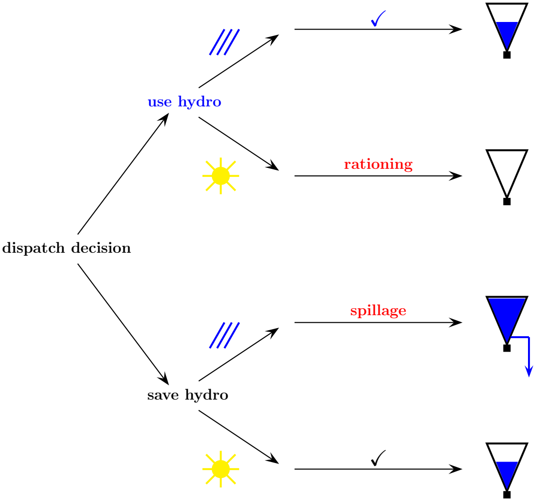
Hydro Scheduling Tradeoff



What is the Problem?

Problem

Decision on the power generation mix (hydro-electric, coal/gas/diesel/bunker fired plants, biomass, etc.) has to be made **today**, taking into account the (non-linear) system characteristics.

Challenge

There is **no monetary value** associated with certain (hydro) reservoir levels!?

Solution

Calculate **future-cost-function** associated with (hydro) reservoir levels through (stochastic) mid-term optimization models.

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Thermal Complement Function

$$c_t(\mathbf{u}_t) := \min \sum_{j \in \mathbb{J}} c_{tj} \mathbf{g}_{tj} + \Upsilon \delta_t \tag{1}$$

$$\text{s.t.} \quad \sum\nolimits_{j \in \mathbb{J}} \mathbf{g}_{tj} + \delta_1 = d_t - \sum\nolimits_{i \in \mathbb{I}} \rho_i \mathbf{u}_{ti} \tag{2}$$

$$\underline{g}_{tj} \leq \mathbf{g}_{tj} \leq \bar{g}_{tj}, \qquad j \in \mathbb{J} \tag{3}$$

$$\mathbf{g}_{tj} \geq 0, \qquad \delta_t \geq 0, \qquad j \in \mathbb{J}. \tag{4}$$

Multi-Stage Stochastic Optimization

$$
\begin{aligned}
z := \min c_1(\mathbf{u}_1) + \min \mathbb{E}_{\omega_2 \in \Omega_2} \Bigg[c_t\big(\mathbf{u}_t(\omega_t)\big) + \ldots + \\
+ \min \mathbb{E}_{\omega_t \in \Omega_t} \Big[c_t\big(\mathbf{u}_t(\omega_t)\big) + \ldots + \\
+ \min \mathbb{E}_{\omega_T \in \Omega_T} \Big[c_T\big(\mathbf{u}_T(\omega_T)\big)\Big] \ldots \Bigg]
\end{aligned}
\tag{5}
$$

Multi-Stage Stochastic Optimization (cont'd)

$$\text{s.t. } \mathbf{v}_{2i} = v_{1i} - \mathbf{u}_{1i} - \mathbf{s}_{1i} + \sum_{h \in \mathbb{U}_i} (\mathbf{u}_{1h} + \mathbf{s}_{1h}) + a_{1i}, \qquad i \in \mathbb{I} \tag{6}$$

$$\begin{aligned} \mathbf{v}_{t+1i}(\omega_t) = \mathbf{v}_{ti}(\omega_{t-1}) - \mathbf{u}_{ti}(\omega_t) - \mathbf{s}_{ti}(\omega_t) + \\ + \sum_{h \in \mathbb{U}_i} \left(\mathbf{u}_{th}(\omega_t) + \mathbf{s}_{th}(\omega_t) \right) + a_{ti}(\omega_t), \qquad t \in \mathbb{T}_1,\ i \in \mathbb{I} \end{aligned} \tag{7}$$

$$\begin{aligned} &\underline{u}_{1i} \le \mathbf{u}_{1i} \le \bar{u}_{1i}, &&\underline{u}_{ti} \le \mathbf{u}_{ti}(\omega_t) \le \bar{u}_{ti}, \\ &\underline{v}_{2i} \le \mathbf{v}_{2i} \le \bar{v}_{2i}, &&\underline{v}_{t+1i} \le \mathbf{v}_{t+1i}(\omega_t) \le \bar{v}_{t+1i}, \\ &\underline{s}_{1i} \le \mathbf{s}_{1i} \le \bar{s}_{1i}, &&\underline{s}_{ti} \le \mathbf{s}_{ti}(\omega_t) \le \bar{s}_{ti}, \qquad t \in \mathbb{T}_1,\ i \in \mathbb{I},\ j \in \mathbb{J} \end{aligned} \tag{8}$$

Is the "Hydro-Thermal Scheduling World" Linear?

No!

...but piecewise linear is a very good approximation!

D.D. Wolf and Y. Smeers
The Gas Transmission Problem Solved by an Extension of the Simplex Algorithm
Management Science, 46, 1454–1465, 2000

R. Rubio-Barros, D. Ojeda-Esteybar, and A. Vargas
Energy Carrier Networks: Interactions and Integrated Operational Planning
Handbook of Networks in Power Systems, P.M. Pardalos, S. Rebennack, M.V.F. Pereira, N. Iliadis, and A. Sorokin (ed.), Springer, to appear

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Solution Methods

Classification with respect to inflow uncertainty methodology:

1. *deterministic* models,
2. *scenario-based* methods,
3. *sampling-based* methods.

W. Yeh
Reservoir management and operations models: A state of the art review
Water Resources Research, 21, 1797–1818, 1985

J. Labadie
Optimal operation of multireservoir systems: State-of-the-art review
Journal of Water Resources Planning and Management, 130, 93–111, 2004

Sampling-Based Methods

Idea

Sampling-based methods generate samples of the random space **on-the-fly** and solve the resulting problems **approximately**.

- typically Dynamic Programming methods
- statistical convergence results
- may possess “Curse of Dimensionality”
- very popular for hydro-thermal scheduling

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Sampling-Based Methods

The major lines of research for sampling-based methods towards hydro-thermal scheduling is driven by the methods of

- **Stochastic Dynamic Programming** (SDP)
- **Stochastic Dual Dynamic Programming** (SDDP)

B.F. Lamond and A. Boukhtouta
Optimizing long-term hydro-power production using markov decision processes
International Transactions in Operational Research, 3, 223–241, 1996

Bellman Recursion: Hydro-Thermal Scheduling

$$z_t(v_t) := \min \mathbb{E}_{\omega\in\Omega_t}\left[\sum_{j\in\mathbb{J}} c_{tj}\mathbf{g}_{tj}(\omega) + \Upsilon\delta_t(\omega) + z_{t+1}\left(\mathbf{v}_{t+1}(\omega)\right)\right] \tag{9}$$

$$\text{s.t. } \sum\nolimits_{j\in\mathbb{J}} \mathbf{g}_{tj}(\omega) + \sum\nolimits_{i\in\mathbb{I}} \rho_i \mathbf{u}_{ti}(\omega) + \delta_t(\omega) = d_t \tag{10}$$

$$\mathbf{v}_{t+1i}(\omega) = v_{ti} - \mathbf{u}_{ti}(\omega) - \mathbf{s}_{ti}(\omega) + \sum\nolimits_{h\in\mathbb{U}_i}\left(\mathbf{u}_{th}(\omega) + \mathbf{s}_{th}(\omega)\right) + a_{ti}(\omega), \quad i\in\mathbb{I} \tag{11}$$

$$\begin{aligned} &\underline{g}_{tj} \le \mathbf{g}_{tj}(\omega) \le \bar{g}_{tj}, \qquad \underline{u}_{ti} \le \mathbf{u}_{ti}(\omega) \le \bar{u}_{ti},\\ &\underline{v}_{t+1i} \le \mathbf{v}_{t+1i}(\omega) \le \bar{v}_{t+1i}, \qquad \underline{s}_{ti} \le \mathbf{s}_{ti}(\omega) \le \bar{s}_{ti},\\ &\delta_t(\omega) \ge 0, \qquad i\in\mathbb{I},\ j\in\mathbb{J}. \end{aligned} \tag{12}$$

Solution Methods

When solving the One Stage Dispatch Problem, one encounters (at least) the following two challenges:

1. the (conditioned) distribution of ω is not known and expected to be continuous, and
2. One Stage Dispatch Problem cannot be solved computationally for the whole continuum of reservoir levels v_t.

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Solution Methods (cont'd)

Stochastic Dynamic Programming (SDP) and **Stochastic Dual Dynamic Programming** (SDDP) overcome these two challenges in the following way:

1. These inflows are modeled as a linear autoregressive model via a continuous Markov Process.
2. The set of reservoir levels is discretized into M values. The function z_t is then approximated either via
   - **interpolation** of the M points (in SDP), or via
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Deterministic One-Stage Programming

$$z_t(v_t, a_{t-1}) := \min \sum_{l \in \mathbb{L}} p^l \left[\sum_{j \in \mathbb{J}} c_{tj} \mathbf{g}_{tj}^l + \Upsilon \delta_t^l + z_{t+1}\left(\mathbf{v}_{t+1}^l, a_t^l\right) \right] \tag{13}$$

$$\text{s.t.} \ \sum\nolimits_{j \in \mathbb{J}} \mathbf{g}_{tj}^l + \sum\nolimits_{i \in \mathbb{I}} \rho_i \mathbf{u}_{ti}^l + \delta_t^l = d_t \tag{14}$$

$$\mathbf{v}_{t+1i}^l = v_{ti} - \mathbf{u}_{ti}^l - \mathbf{s}_{ti}^l + \sum\nolimits_{h \in \mathbb{U}_i} \left(\mathbf{u}_{th}^l + \mathbf{s}_{th}^l\right) + a_{ti}^l, \quad i \in \mathbb{I} \tag{15}$$

$$\begin{aligned} &\underline{g}_{tj} \le \mathbf{g}_{tj}^l \le \bar{g}_{tj}, \qquad \underline{u}_{ti} \le \mathbf{u}_{ti}^l \le \bar{u}_{ti}, \\ &\underline{v}_{t+1i} \le \mathbf{v}_{t+1i}^l \le \bar{v}_{t+1i}, \qquad \underline{s}_{ti} \le \mathbf{s}_{ti}^l \le \bar{s}_{ti}, \\ &\delta_t^l \ge 0, \qquad i \in \mathbb{I}, j \in \mathbb{J}, l \in \mathbb{L}, \end{aligned} \tag{16}$$

SDDP: Expected Future Cost **Extrapolation**

- use information of dual to **underestimate** future cost function
- "Benders cuts"
- backwards pass: $\underline{z}$
- forward Monte Carlo simulation: $\hat{z}$
- stop when convergence criteria satisfied

$$\hat{\sigma} := \sqrt{\frac{1}{M-1} \sum_{m \in \mathbb{M}} (z^m - \hat{z})^2}. \tag{17}$$

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SDDP: Strength

1. **no** curse of dimensionality
2. state space is discretized **dynamically**
3. statistical solution quality measure

M.V.F. Pereira
Optimal stochastic operations scheduling of large hydroelectric systems
International Journal of Electrical Power & Energy Systems, 11, 161–169, 1989

M.V.F. Pereira and L.M.V.G. Pinto
Multi-stage stochastic optimization applied to energy planning
Mathematical Programming, 52, 359–375, 1991

SDDP: Extensions, Variations and Related Methods

- Convergence Analysis; Philpott, Shapiro
- Abridged Nested Decomposition (AND); Birge
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- Generalized Dual Dynamic Programming (GDDP); Velásquez
- Constructive Dual Dynamic Programming (CDDP); Read
- Hybrid SDP/SDDP; Gjelsvik

SDDP is established method and state-of-the-art. It is used in more than 30 countries spread across 5 continents.

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CO2 Emission Constrained SDDP

Introduction: Global Warming

Dr. John Marburger,
G.W. Bush's chief scientific adviser:

It is more than 90 percent certain that greenhouse gas emissions to blame for rising global temperatures.

BBC News, September 14, 2007

Emissions: New Challenges

Emission Quotas: *Policy Makers*

1. How to define a meaningful *quota level* for an energy system?
2. What are the *effects* (economic + environmental) of such a quota?
3. What are the *operational* consequences?

Emission Markets: *Utilities*

4. How to *optimize* with respect to stochastic CO2 prices?
5. Can we *predict* stochastic CO2 prices?
6. How to deal with the *correlation* of the hydro-system, fuel prices and CO2 prices?

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Least-Cost Hydro-Thermal Scheduling

- Considering a “closed system” for CO2 emissions
- No trading of emissions possible
- Given CO2 emission quota; penalty fee has to be paid if quota is exceeded
- CO2 emissions allowances are issued per term to avoid random anomalies
- (All market participants are price takers)

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CO2 Allowances Modeling

The CO2 allowances can then be modeled as follows

$$\sum_{t|y} \sum_{j \in \mathbb{J}} B_j \mathbf{g}_{tj}(\omega) - \mathbf{f}_y(\omega) \leq E_y^{\mathrm{CO2}}, \quad y \in \mathbf{Y}_g$$

where $y \in \mathbf{Y}_g \subseteq \mathbb{T}$ is the set of stages when the CO2 allowances are issued.

SDDP?

Does **not** work in a "one-stage" framework of SDP/SDDP.

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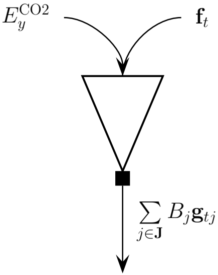
CO2 Emission Allowances Modeling via Reservoirs



$$\mathbf{e}_{t+1} = e_t - \sum_{j \in \mathbb{J}} B_j \mathbf{g}_{tj} + \mathbf{f}_t, \quad t \in \mathbf{T} \setminus \mathbf{Y}_g \tag{18}$$

$$\mathbf{e}_{t+1} = \widetilde{e}_t - \sum_{j \in \mathbb{J}} B_j \mathbf{g}_{tj} + \mathbf{f}_t + E_t^{\mathrm{CO2}}, \quad t \in \mathbf{Y}_g \tag{19}$$

$$\mathbf{e}_{t+1} \geq 0, \quad \mathbf{f}_t \geq 0, \quad t \in \mathbb{T}, \tag{20}$$

with

$$\widetilde{e}_t := \begin{cases} 0, & \text{if the emissions expire} \\ e_t, & \text{if the emissions do not expire} \end{cases}, \quad t \in \mathbf{Y}_g \tag{21}$$

Future Cost Function Cuts

Evaluating function z_t at a specific point ν_t^n, e_t^n and a_{t-1}^n leads to a function value $z_t(\nu_t^n, e_t^n, a_{t-1}^n) \in \mathbb{R}$.

Function z_t is **convex** in ν_t^n, e_t^n and a_{t-1}^n.

If we know also the slopes γ_{tn}^ν, γ_{tn}^e and γ_{tn}^a of z_t at this point ν_t^n, e_t^n and a_{t-1}^n, then we can **extrapolate** the function z_t.

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Future Cost Function Cuts

Hence, we can *underestimate* the function z_t via the (linear) slopes of the points ν_t^m, e_t^n and a_{t-1}^n and the following linear program:

$$\underline{z}_t = \min \alpha \tag{22}$$

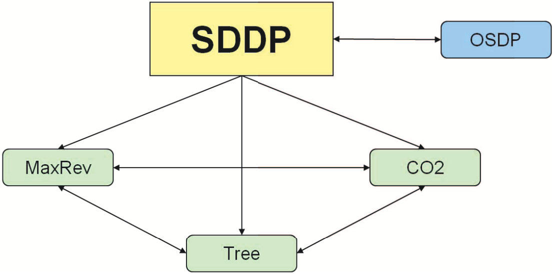
$$\text{s.t. } \alpha \geq \gamma_{tn}^{\nu} \nu_t^n + \gamma_{tn}^{e} e_t^n + \gamma_{tn}^{a} a_{t-1}^n + \gamma_{tn}^{c}, \quad n \in \mathbb{N} \tag{23}$$

where $n \in \mathbb{N} = \{1, \ldots, N\}$ denoted the n-th linear segment of the convex underestimation and γ_{tn}^c is the corresponding constant term.

Famework

System:

- Standard laptop
- XPRESS Mosel, XPRESS-MP version 20.00
  $\approx$ 5000 lines (including comments)



Case Study: Guatemala

One hydro reservoir with a water storage capacity of 440 hm^3 and an installed capacity of 275 MW.

Table: Thermal plants considered for the Guatemala power system

Number of Plants	1	3	1	18	3
Cumulative Capacity [MW]	24.0	120.4	41.4	729.8	91.5
Fuel Type	1	1	2	2	2
Cost [$/MWh]	129.9	132.0	61.6	67.1	68.7
CO2 Emission [kg/MWh]	625.0	635.2	544.1	593.5	607.3
Number of Plants	1	1	3	10	
Cumulative Capacity [MW]	132.4	13.0	58.0	227.0	
Fuel Type	3	3	4	5	
Cost [$/MWh]	41.2	45.9	2.7	1.0	
CO2 Emission [kg/MWh]	1001.0	1115.4	0	0	

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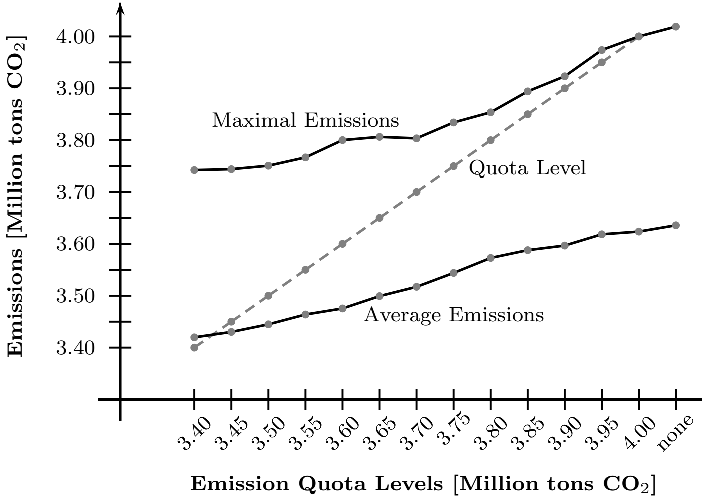
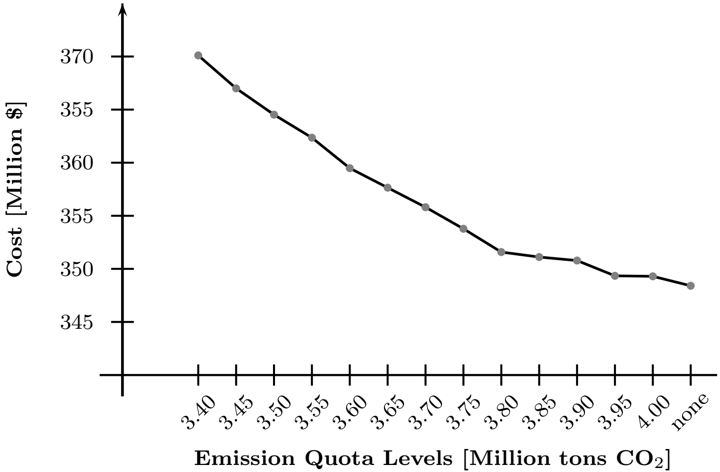
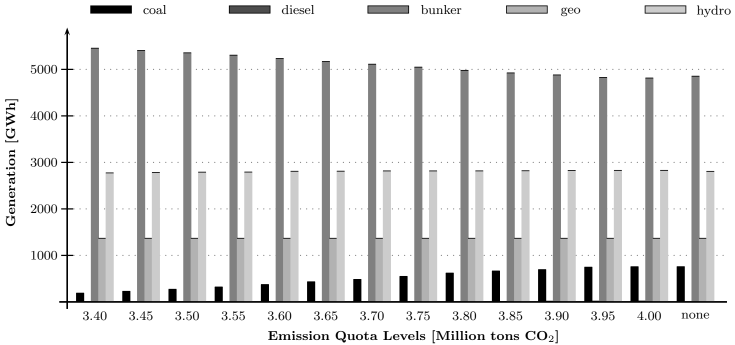
Annual CO2 Emissions



Annual Operational Cost



Yearly Generation Mix



Monthly Dispatching

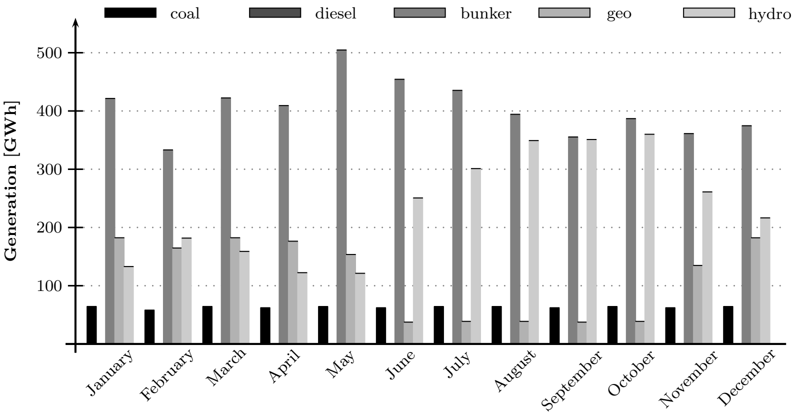


Figure: Monthly dispatching decisions for the quota free case

Monthly Dispatching (cont'd)

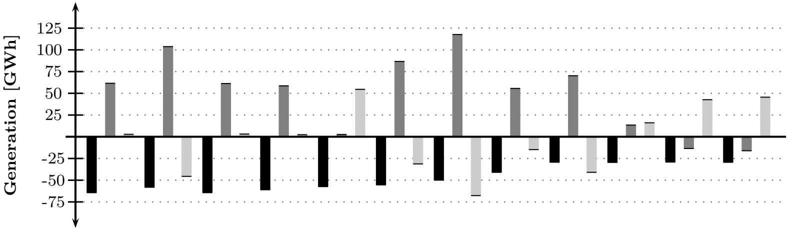


Figure: Monthly dispatching decisions with quota of 3.40 Million tons; relative to quota free case monthly difference in electricity

Monthly Dispatching (cont'd)

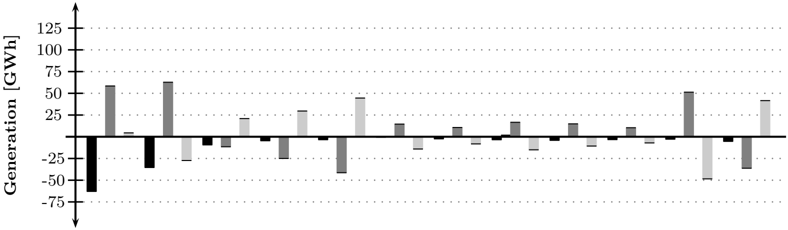
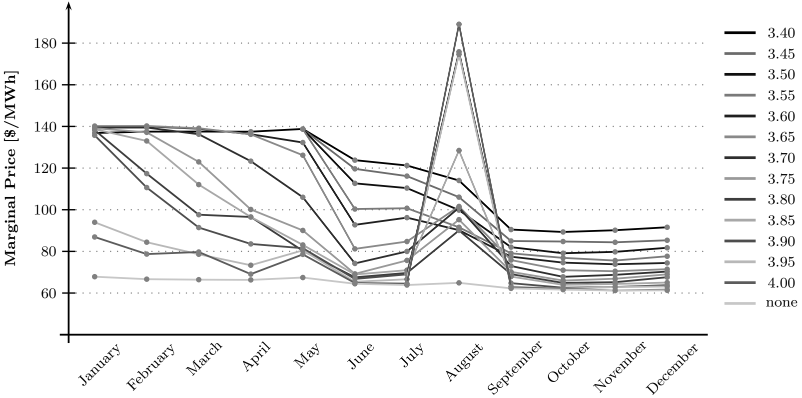
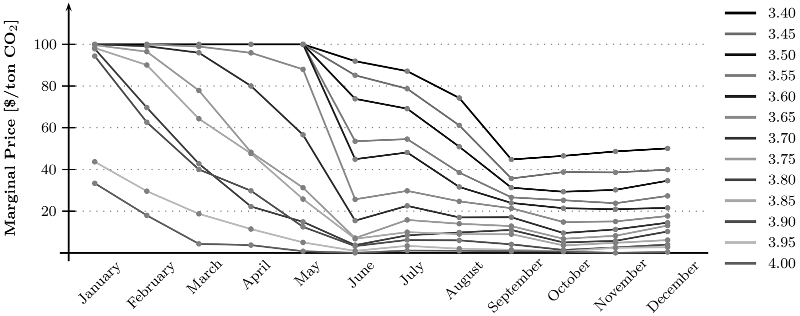


Figure: Monthly dispatching decisions with quota of 3.80 Million tons; relative to quota free case monthly difference in electricity

Average Electricity Marginal Prices



Average CO2 Emission Allowance Marginal Prices



Conclusions

1. Introduction
2. CO2 Emission Constrained SDDP
3. Conclusions
   - Conclusions
   - Future Work
   - Discussion

Conclusions

1. Meaningful quota levels. ✓
2. Effects of quota. ✓
3. Operational consequences. ✓

Main Contribution

Modeling of CO2 emission quota respecting the stage decomposition framework of SDDP

S. Rebennack, B. Flach, M.V.F. Pereira, and P.M. Pardalos
Hydro-Thermal Scheduling under CO2 Emission Constraints
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Future Work

1. Application to *Optimal Expansion Planning* (ongoing).
2. *Clustering Techniques* for the electricity spot prices and CO2 emission allowance market prices (ongoing).
3. Incorporation of risk measures in the models.
4. Extension to non-linear models.

Conference

SEA2011 - 10th International Symposium on Experiential Algorithms

Chania, Creete, Greece

Panos M. Pardalos and Steffen Rebennack

Important Dates:

Full Paper Submission Deadline: January 21st, 2011

Opening Cocktail: May 4th, 2011

Conference: May 5-7th, 2011

The END!

Questions, Comments, Suggestions?